

Kinematics of Circular Motion  
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Introduction

*Part 1*

Circular motion (or any angular motion) is non-inertial. That is to say that because movement is not in a straight line at constant speed, there must be a force acting to change the direction of objects moving in such a manner. Often this force is centripetal (meaning pointing to the center of the circle). The centripetal force is the product of the centripetal acceleration, given by:

$$1) \quad a_c = \frac{v^2}{r}$$

and the mass,  $m$ . If the object is truly moving in a circle, the velocity is given by the circumference,  $C$ , divided by the period,  $T$ , or time it takes to go around once.

$$2) \quad v = \frac{C}{T} = \frac{2\pi r}{T}$$

By combining equations 1) and 2), one can get the following expression for centripetal acceleration:

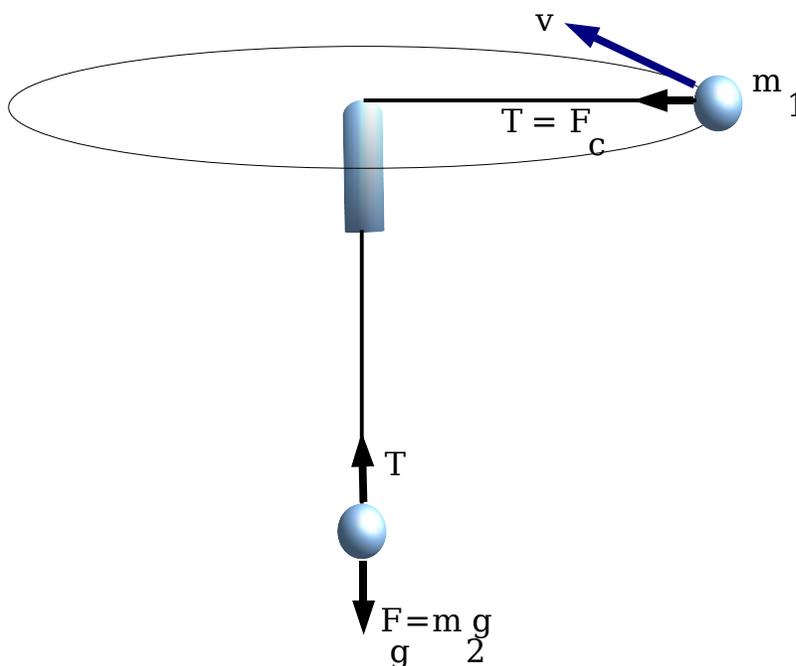
$$3) \quad a_c = \frac{4\pi^2 r}{T^2}$$

In the first portion of this lab, you will be balancing the centripetal force caused by spinning a small mass,  $m_1$ , around on a fixed length string with the gravitational force of a hanging mass,  $m_2$ . Setting these forces equal to each other, we have the following relationship:

$$4) \quad m_2 g = \frac{4\pi^2 m_1 r}{T^2}$$

Finally, by rearranging equation 4), we can get an expression for period in terms of everything else:

$$5) \quad T = \sqrt{\frac{4\pi^2 m_1 r}{m_2 g}}$$



*Drawing 1: Setup for Part 1*

## Part 2

In the second part of this lab, you will be calculating the angular position, angular velocity and angular acceleration of video-taped motion.

A motion capture program will be demonstrated so that you understand where this data came from. You will be presented with data in the format of a p2d file. This lists a marker name and then the time information, and the x and y coordinates that correspond to the time:

### Marker 1

<i>time (s)</i>	<i>x-coord (pixels)</i>	<i>y-coord (pixels)</i>
0.000	547	124
0.010	556	118
0.020	565	109
...		

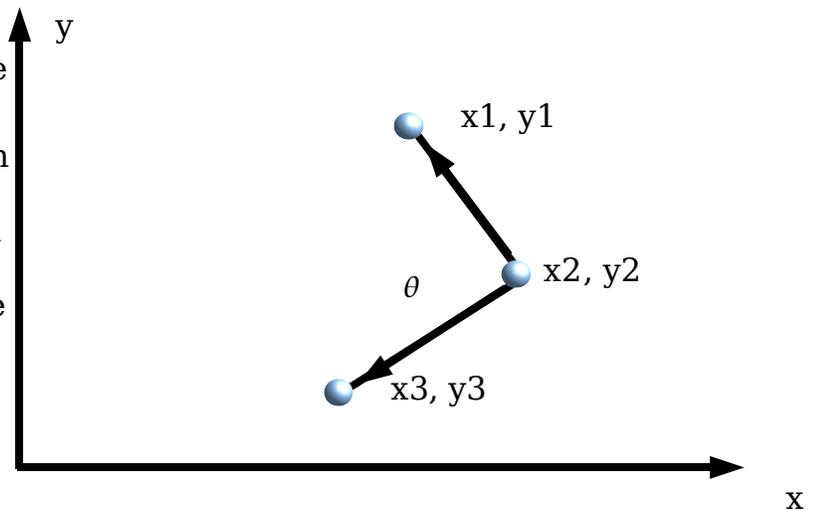
### Marker 2

<i>time (s)</i>	<i>x-coord (pixels)</i>	<i>y-coord (pixels)</i>
0.000	627	224
0.010	626	218
0.020	625	209
...		

### Marker 3

<i>time (s)</i>	<i>x-coord (pixels)</i>	<i>y-coord (pixels)</i>
0.000	507	134
0.010	516	178
0.020	525	189
...		

The diagram at the right illustrates one frame of data for the three markers. Our goal is to find the angle  $\theta$ , which lies between the vector from point 2 to point 1, and the vector from point 2 to point 3. It is easiest to proceed by finding the angle between this first vector and the x-axis, and then by finding the angle between the second vector and the x-axis. The angle  $\theta$  is the difference between these measures.



$$6) \theta_1 = \text{atan2}\left(\frac{y_1 - y_2}{x_1 - x_2}\right)$$

and

$$7) \theta_2 = \text{atan2}\left(\frac{y_3 - y_2}{x_3 - x_2}\right)$$

Thus:

$$8) \theta = \theta_2 - \theta_1$$

This calculation must be done for every frame of data.

## Procedure

### Part 1

Obtain two masses of differing weights, a PVC tube, a string, and a stop watch. Thread the string through the tube, and firmly attach the larger of the masses so that it hangs down as shown in Drawing 1. Attach the smaller of the masses to the other end of the string *after* it has been thread through the tube. This is the end that will swing around.

While one partner controls the stop watch, the other should start by slowly spinning the small mass in a horizontal plane about his head. Spin the mass at such a rate that the hanging mass does not either rise or fall. Measure the time it takes to complete ten full cycles and determine the period, T, of rotation. Measure the length of string from the tube to the mass. Then repeat but at a different angular velocity and length. Repeat at least ten times with different lengths and velocities.

### Part 2

The procedure for this part has been attenuated due to time constraints. You will simply analyze data that has already been collected for you on a soccer player kicking a ball.

## Analysis

### Part 1

Plot T versus the *square root of r* for your data. Fit a line to the data. Use the excel function "LINEST" to determine the slope and uncertainty in the slope. For example, presume that your data for r (the x variable) is in the range A1:A10 and that B1:B10, you would use "LINEST" as follows:

<i>Excel</i>	<i>OpenOffice</i>
{=LINEST(B1:B10,A1:A10,TRUE,TRUE)}	{=LINEST(B1:B10;A1:A10;1;1)}

**Instead of hitting enter after typing this, press CTRL-SHIFT-ENTER.** If you do not do this your results will look very different.

You will see output as follows (but with actual numbers in place):

<b>m (slope)</b>	b (intercept)
<b>standard error of m (SE<sub>m</sub>)</b>	standard error of b
R <sup>2</sup> (correlation coefficient)	standard error of regression
F (a statistic)	degrees of freedom
SS <sub>reg</sub> (sum of squared regression)	SS <sub>resid</sub> (sum of squared residuals)

We are interested in the slope and its standard error which we shall use as its uncertainty. If all is well, your slope should be equal to:

$$9) \quad slope = \sqrt{\frac{4\pi^2 m_1}{m_2 g}}$$

If your measurements validate the theory of centripetal force being balanced by gravitational force (and the overall theory of centripetal motion), the following will be true:

$$10) \quad \left| slope_{excel} - \sqrt{\frac{4\pi^2 m_1}{m_2 g}} \right| < SE_m$$

Use your measured values of mass for the second term.

Comment on the validity of your data. What sort of factors could contribute to the uncertainty in your measurements? Are there scenarios in which the above theory of centripetal motion would be invalidated? If so, describe them.

*Part 2*

Using the data set given in lab, use Excel to calculate the angle, angular velocity, and angular acceleration for each frame. You will use the method shown in the introduction to find the angle in degrees. Next, calculate the angular velocity using the finite difference method introduced in lab 1. Plot both the angles and the angular velocities versus time on separate graphs. Comment on how reasonable your answers are knowing the activity portrayed. If you knew that the length of the lower leg was 0.50 m, what would its fastest linear velocity be when measured with respect to the knee?

