Projectile Motion

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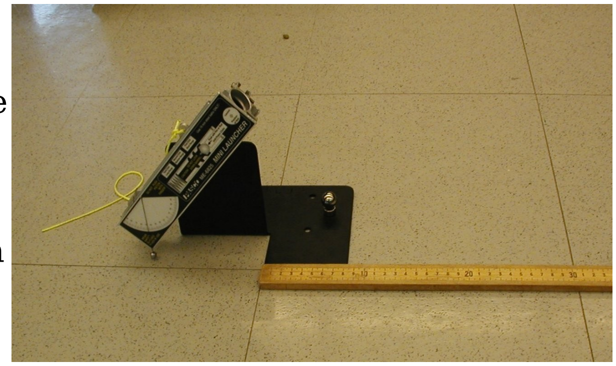
Introduction



Motion in two dimensions follows the same rules as motion in one dimension in that the same kinematics equations are applied. It is important to note, however, that motions in the vertical direction are independent of motions in the horizontal direction. We can thus write two independent sets of kinematic equations, one set for each dimension.

Horizontal	***Vertical***
1a) $x = x_o + v_{ox}t + a_xt^2/2$	1b) $y = y_o + v_{oy}t + a_yt^2/2$
2a) $v_{ox} = v_o\cos\theta$	2b) $v_{oy} = v_o\sin\theta$
3a) $v_x = v_{ox} + a_xt$	3b) $v_y = v_{oy} + a_yt$

In the absence of air resistance, there is no x-component of acceleration, a_x, in projectile motion. This leads to the simplification of the horizontal equations:

4) $x = x_o + v_{ox}t = x_o + v_o\cos\theta\ t$

5) $v_{ox} = v_o\cos\theta$

6) $v_x = v_{ox}$

Equation 4a) allows one to predict the final landing position of the projectile given the time of flight, the angle of inclination of the projectile, and the initial projectile velocity.

The key question is how to determine each of these variables. The angle of inclination is easily obtained via the protractor on the launcher. Unfortunately, the time of flight and the initial velocity are not so obviously obtained. Here is where we must make use of the vertical kinematic equations 1b-3b. By launching the projectile vertically and measuring the maximum height to which the projectile goes, one can estimate the initial velocity. This can be given by the equation:

7) $$v_o = v_{oy} = \sqrt{2gy}$$

Finally, we must try to obtain the time of flight for the general case where we might not be able to measure the maximum height of the projectile. This is a bit more complicated. Inverting equation 7) to solve for y, we get

8) $y=\frac{v_{oy}^2}{2g}$

Equation 1b) can be manipulated to give the half-flight time:

9) $t_{1/2}=\sqrt{\frac{2y}{g}}$

Substituting equation 8) into equation 9), we get an expression for the half-flight time in terms of known quantities:

10) $t_{1/2}=\sqrt{\frac{2(v_{oy}^2/(2g))}{g}}$

or, with some simplification:

11) $t_{1/2}=\frac{v_{oy}}{g}=\frac{v_o \sin\theta}{g}$

Thus the full flight time is equation 11) times two:

12) $t=2\frac{v_o \sin\theta}{g}$

Finally, we have an expression for the expected horizontal distance of travel in terms of all known quantities:

13) $x=x_o+\frac{2v_o^2 \cos\theta \sin\theta}{g}$

Procedure

Part 1: Determining v_o

As indicated by the above discussion, we must first find v_o before we can proceed to predict the range of the projectile as a function of angle. To do this, we will aim the launcher vertically upwards and attempt to measure the height of the ball's apex. Clamp your ruler vertically adjacent to the launcher. Prime your launcher by setting the ball in to the first setting. (There are three. We want the lowest velocities to avoid accidents between groups or with the lights!) One partner should fire the launcher while the other partner observes the ruler to determine the maximal height. Be sure to account for the height difference between the point where the ball leaves the launcher and the apex. Then use equation 7) to calculate the initial velocity. Repeat this ten times to ensure reliability of your measurement. Do not forget to record uncertainties with every measurement.

Table 1: Determination of Initial Velocity

Trial	Muzzle Height y_0 (m)	Apex Height y_1 (m)	Difference in Height, y (m)	Initial Velocity v_o (m/s)
1				
2				
3				
4				
5				
6				
7				
8				
9				
10				
			Average v_o =	
			STDev v_o =	

Part 2: Validation of equation 13)

In this part you will launch the ball at several different angles compare your prediction of the range of the ball with the actual distance traveled. Because the launchers are not level with the ground, you will need to determine how to modify equation 13) to account for this. Otherwise, your predicted range will under-estimate your measured range. Do not forget to record uncertainties for every measured and calculated value.

Table 2: Comparison of Measured vs. Predicted Range

Trial	Angle θ (degrees)	Measured Range, Δx_m (m)	Predicted Range, Δx_m (m)	Within Uncertainty?
1	10			
2	20			
3	30			
4	40			
5	50			
6	60			
7	70			
8	80			

Analysis

How did you determine v_o? (Did you use an average or weighted average.) Why did you choose the method you did? How reliable was the launcher in shooting the ball to the same height? What sources of experimental uncertainty are there in the determination of v_o?

Discuss how you determined the modifications to equation 13) for use in Table 2. Were you able to predict the range reliably? What sources of uncertainty are there in this portion of the lab? What did you do to minimize such uncertainty in your measurements?

Create a graph of range versus angle of launch and discuss what this indicates. Does the maximum range correspond to what you would predict? Do your results validate the independent usage of kinematics equations in two dimensions?