Harmonic Motion and Hooke's Law

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Introduction

Hooke's Law relates the elongation of a spring to its restoring force. If the elongation of a spring is given by Δx, and the stiffness of the spring is given as k, the force exerted by the spring against the mass, m, is given by ma:

1) $ma - k\Delta x = 0$

If the mass is set in motion, it will oscillate with a frequency that is related to the mass and the spring constant:

2) $$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

The period of the oscillation is the reciprocal of the frequency:

3) $$T = 2\pi\sqrt{\frac{m}{k}}$$

Because it is possible to measure the period to a fairly high precision with good experimental technique, if one knows the stiffness, k, of the spring, it is possible to predict the mass that is oscillating at the end of the spring.

Procedure

Part 1: Determining the stiffness

Set up the experiment as indicated in Illustration 1. Measure the resting length of the spring. Next, attach the 10 g mass to the end of the spring. Measure the new length of the spring. Repeat with the remaining masses up to 500 g.

***m* (kg)**	***x_i* (m)**	***$\Delta x = x_{i+1} - x_i$* (m)**
0 (rest length)		
0.010		
0.020		
0.030		
0.040		
0.050		
0.100		
0.150		
0.200		
0.300		
0.400		
0.500		

Part 2: Validating equation 3)
Connect a mass to the spring and gently elongate the spring. Release and let the mass oscillate. Using the provided stopwatch, time the length of the period (use the method of timing 10 oscillations and divide by ten). Record the mass used and the length of the period. Also measure the period of an unknown mass. You will later predict this mass from equation 3). Be sure to mass this unknown on the scale.

***m* (kg)**	***10T* (s)**	***T* (s)**
0.100		
0.200		
0.300		
0.400		
0.500		
unknown mass		

Analysis

Using the data from part 1, plot mg versus Δx. Fit a line to this data (be sure that you have the proper values on the x and y axes). The slope of this line corresponds to the stiffness, k. Estimate the uncertainty in this value.

For part 2, plot T^2 vs m and fit an equation to determine the slope. Use this slope to estimate k. Is this k the same as that from part 1 within experimental uncertainty? If not, why not? Use the k from part 2 to estimate the unknown mass. Is your estimate of the unknown the same as your actual measurement on the scale?

What conclusions can you draw from your results regarding the validity of equation 3) and the theory of simple harmonic motion as applied to a spring?